ber and thickness ratio on the damping-in-pitch normal force and moment coefficient slopes of a parabolic spindle. Although the two theories predict similar trends with Mach number, there is a difference in absolute levels in each case which may be caused by the difference in coupling with the mean flow and/or by Revell's inclusion of the quadratic frequency terms.

IV. Summary

A theory was developed for subsonic flow past slowly oscillating pointed bodies of revolution. By properly expanding the first-order velocity potential, correction terms to slender-body theory could be derived which account for thickness and compressibility effects. This work is an extension of previous approaches by Adams and Sears[3], Laitone,[1] and Schultz-Piszachich[2] for flow past bodies at steady angle of attack, and the equivalence of these approaches is pointed out. The numerical results show good agreement between Revell's second-order slender-body theory and the present theory for the static stability derivatives of parabolic spindles. Thus it gives confidence in the use of this much simpler theory for slender bodies. Further experiments are required to assess the reliability of the dynamic stability predictions of both theories.

References

[1] Laitone, E. V., "The Subsonic Flow About a Body of Revolution," *Quarterly of Applied Mathematics,* Vol. 5, 1947, pp. 227-234.

[2] Schultz-Piszachich, W., "Beitrag zur Formelmassigen Berechung der stationaren Geschwindigkeitsverteilung umstromter Drehkorper im Unter-und Uberschallbereich," *Oesterreichisches Ingenieur Archiv,* Vol. 5, 1951, pp. 289-303.

[3] Adams, M. C. and Sears, W. R., "Slender-Body Theory—Review and Extension," *Journal of the Aeronautical Sciences,* Vol. 20, Feb. 1953, pp. 85-98.

[4] Keune, F., "Reihenentwicklung des Geschwindigkeitspotentials der linearen Unter-und Ueberschallstromung fur Korper nicht mehr kleiner Streckung," *Zeitschrift fur Flugwissenschaften,* Vol. 5, Nov. 1957, pp. 243-247.

[5] Hess, J. L. and Smith, A. M. O., "Calculation of Non-Lifting Potential Flow About Arbitrary Three Dimensional Bodies," *Journal of Ship Research,* Vol. 8, June 1964, pp. 15-23.

[6] Revell, J. D., "Second-Order Theory for Steady or Unsteady Subsonic Flow Past Slender Lifting Bodies of Finite Thickness," *AIAA Journal,* Vol. 7, June 1969, pp. 1070-1078.

[7] Platzer, M. F. and Hoffman, G. H., "Quasi-Slender Body Theory for Slowly Oscillating Bodies of Revolution in Supersonic Flow," NASA TND-3440, June 1966.

[8] Revell, J. D., "Second-Order Theory for Unsteady Supersonic Flow Past Slender Pointed Bodies of Revolution," *Journal of the Aerospace Sciences,* Vol. 27, Oct. 1960, pp. 730-740.

[9] Liu, D. D. and Platzer, M. F., "Sonic and Subsonic Flow Past Slowly Oscillating Bodies of Revolution," Rept. ER-10221, Oct. 1969, Lockheed-Georgia Co.

[10] Miles, J. W., *Theory of Unsteady Supersonic Flow,* Cambridge University Press, Cambridge, England, 1959.

[11] Liu, D. D., Platzer, M. F., and Ruo, S. Y., "On the Calculation of Static and Dynamic Stability Derivatives for Bodies of Revolution at Subsonic and Transonic Speeds," AIAA Paper 70-190, Jan. 19-20, 1970.

Governing Equations for Large Deflections of Sandwich Plates

N. Kamiya*
MIE University, Kamihamacho, Tsu, Japan

Nomenclature

a,b = linear dimensions of rectangular sandwich plate
E = Young's modulus of face
$F(b/a)$ = correction function, Eq. (27)
G' = shear modulus of core
h = core thickness (measured from middle surfaces of each face)
I_1^m, I_2^m = first and second invariants of averaged strains
q = lateral load
t = face thickness
u,v,w = displacements in x, y, and z directions, respectively
$\bar{V}_0$ = strain energy per unit area
x,y,z = rectangular coordinate system
ϵ,γ = strain components
σ,τ = stress components
ν = Poisson's ratio of face
$(\ )^c$ = core variable
$(\ )^f$ = face variable
$(\ )^u$ = upper face variable
$(\ )^\ell$ = lower face variable
$(\ )^m$ = averaged value

Received June 20, 1975; revision received September 3, 1975.
Index category: Structural Static Analysis.
*Assistant Professor Mechanical Engineering.

Introduction

MANY investigations have been carried out on finite deformations of sandwich plates and shells, which have practical importance.[1-6] About two decades ago, Berger[7] proposed an interesting approximate method for large deflection problems of single-layer isotropic plates. His method is based on unqualified disregard of a second invariant of middle plane strains of plate (membrane strains) in the strain energy formula, and then by the variational calculus linear system of equations were derived with respect to displacements, in which the deflection is decoupled with inplane displacements. This idea was extended to static and dynamic problems of anisotropic as well as isotropic plates and shallow shells.[8-10]

However, so far as we know, no application of it to sandwich plates and shells has been reported. Since deformation responses of faces and core of sandwich structures are different, it is only natural that Berger's original idea may not be directly applicable. In this Note, we deal with an average of fiber strains of each face, which is algebraically identical to the membrance strain of conventional plates, and then we apply the foregoing Berger's method to a symmetrically-laminated sandwich plate with large deflection. Numerical illustrations of laterally loaded rectangular sandwich plates are presented.

Governing Equations

First, we posit a rectangular coordinate system x, y, z: x, y in the middle plane of the core, z thickness direction (positive downward). For the sake of simplicity, consider a sandwich plate with an isotropic core as well as isotropic upper and lower faces of identical thickness. While the faces respond to the bending and membrance actions of the plate, the core is assumed to transfer only shear deformations. Moreover, compared with the core thickness h, the face thickness t is supposedly thin enough to ignore a variation of stress in the thickness direction of the faces.

The strain components of each face are expressed, in terms of each displacement components, as

$$\epsilon_x^u = \frac{\partial u^u}{\partial x} + \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \epsilon_y^u = \frac{\partial v^u}{\partial y} + \tfrac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy}^u = \frac{\partial u^u}{\partial y} + \frac{\partial v^u}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial x}, \tag{1a}$$

$$\epsilon_x^\ell = \frac{\partial u^\ell}{\partial x} + \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \epsilon_y^\ell = \frac{\partial v^\ell}{\partial y} + \tfrac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy}^\ell = \frac{\partial u^\ell}{\partial y} + \frac{\partial v^\ell}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial x} \tag{1b}$$

Here, we introduce averaged values of both face strain components,

$$\epsilon_x^m = \tfrac{1}{2}(\epsilon_x^u + \epsilon_x^\ell),\ \epsilon_y^m = \tfrac{1}{2}(\epsilon_y^u + \epsilon_y^\ell),\ \gamma_{xy}^m = \tfrac{1}{2}(\gamma_{xy}^u + \gamma_{xy}^\ell) \qquad (2)$$

With the aid of Eqs. (2), Eqs. (1) become

$$\epsilon_x^u = \epsilon_x^m + \tfrac{1}{2}\frac{\partial r}{\partial x},\ \epsilon_y^u = \epsilon_y^m + \tfrac{1}{2}\frac{\partial s}{\partial y},$$

$$\gamma_{xy}^u = \gamma_{xy}^m + \tfrac{1}{2}\frac{\partial r}{\partial y} + \tfrac{1}{2}\frac{\partial s}{\partial x} \qquad (3a)$$

$$\epsilon_x^\ell = \epsilon_x^m - \tfrac{1}{2}\frac{\partial r}{\partial x},\ \epsilon_y^\ell = \epsilon_x^m - \tfrac{1}{2}\frac{\partial s}{\partial y},$$

$$\gamma_{xy}^\ell = \gamma_{xy}^m - \tfrac{1}{2}\frac{\partial r}{\partial y} - \tfrac{1}{2}\frac{\partial s}{\partial x} \qquad (3b)$$

where the following notations are used

$$r = u^u - u^\ell,\quad s = v^u - v^\ell \qquad (4)$$

By virtue of Hooke's law for isotropic elastic materials, the strain energy per unit area of the both faces is represented as

$$\bar{V}_0{}^f = \frac{Et}{1-\nu^2}\Big[\epsilon_x^{m2} + \tfrac{1}{4}\Big(\frac{\partial r}{\partial x}\Big)^2 + \epsilon_y^{m2} + \tfrac{1}{4}\Big(\frac{\partial s}{\partial y}\Big)^2$$

$$+ 2\nu\Big(\epsilon_x^m\epsilon_y^m + \tfrac{1}{4}\frac{\partial r}{\partial x}\frac{\partial s}{\partial y}\Big) + \frac{1-\nu}{2}\Big\{\gamma_{xy}^{m2}$$

$$+ \tfrac{1}{4}\Big(\frac{\partial r}{\partial y}\Big)^2 + \tfrac{1}{4}\Big(\frac{\partial s}{\partial x}\Big)^2 + \tfrac{1}{2}\frac{\partial r}{\partial y}\frac{\partial s}{\partial x}\Big\}\Big] \qquad (5)$$

Furthermore, if we rewrite Eq. (5) by introducing two invariants of the averaged strains

$$I_1^m = \epsilon_x^m + \epsilon_y^m,\quad I_2^m = \epsilon_x^m\epsilon_y^m - \tfrac{1}{4}\gamma_{xy}^{m2} \qquad (6)$$

we obtain

$$\bar{V}_0{}^f = \frac{Et}{1-\nu^2}\Big[I_1^{m2} - 2(1-\nu)I_2^m + \tfrac{1}{4}\Big(\frac{\partial r}{\partial x}\Big)^2 + \tfrac{1}{4}\Big(\frac{\partial s}{\partial y}\Big)^2$$

$$+ \frac{\nu}{2}\frac{\partial r}{\partial x}\frac{\partial s}{\partial y} + \frac{1-\nu}{8}\Big(\frac{\partial r}{\partial y} + \frac{\partial s}{\partial x}\Big)^2\Big] \qquad (7)$$

Since the shear strains of the core may be expressed as

$$\gamma_{xz} = \frac{1}{h}(u^\ell - u^u) + \frac{\partial w}{\partial x},\quad \gamma_{yz} = \frac{1}{h}(v^\ell - v^u) + \frac{\partial w}{\partial y} \qquad (8)$$

the strain energy per unit area of the isotropic core due to the shear becomes

$$\bar{V}_0{}^c = \tfrac{1}{2}hG'\Big[\Big(\frac{r}{h}\Big)^2 + \Big(\frac{s}{h}\Big)^2 + \Big(\frac{\partial w}{\partial x}\Big)^2 + \Big(\frac{\partial w}{\partial y}\Big)^2$$

$$- \frac{2}{h}\Big(r\frac{\partial w}{\partial x} + s\frac{\partial w}{\partial y}\Big)\Big] \qquad (9)$$

In consequence, the total strain energy per unit area of the sandwich plate is

$$\bar{V}_0 = \bar{V}_0^f + \bar{V}_0^c \qquad (10)$$

According to Berger's line of thought, we ignore I_2^m appearing in Eqs. (7) and (10). Executing the variational calculus so as to minimize the total potential energy of the present elastic system of the sandwich plate, after lengthy calculations, we arrive at the following five equations

$$\partial I_1^m/\partial x = 0 \qquad (11)$$

$$\partial I_1^m/\partial y = 0 \qquad (12)$$

$$\frac{Et}{2(1-\nu^2)}\Big(\frac{\partial^2 r}{\partial x^2} + \nu\frac{\partial^2 s}{\partial x\partial y}\Big) + \frac{Et}{4(1+\nu)}\Big(\frac{\partial^2 r}{\partial y^2}$$

$$+ \frac{\partial^2 s}{\partial x\partial y}\Big) - G'\Big(\frac{r}{h} - \frac{\partial w}{\partial x}\Big) = 0 \qquad (13)$$

$$\frac{Et}{2(1-\nu^2)}\Big(\frac{\partial^2 s}{\partial y^2} + \nu\frac{\partial^2 r}{\partial x\partial y}\Big) + \frac{Et}{4(1+\nu)}\Big(\frac{\partial^2 s}{\partial x^2}$$

$$+ \frac{\partial^2 r}{\partial x\partial y}\Big) - G'\Big(\frac{s}{h} - \frac{\partial w}{\partial y}\Big) = 0 \qquad (14)$$

$$\Big(\frac{2Et}{1-\nu^2}I_1^m + G'h\Big)\nabla^2 w - G'\Big(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\Big) + q = 0 \qquad (15)$$

where

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

In the previous derivation external and boundary force terms except for lateral load q are thoroughly excluded for abbreviation. Thus derived equations are approximate governing equations for large deflections of sandwich plates in the context of a reduced Berger's method. According to Eqs. (11) and (12), I_1^m is constant, which result is analogous to Berger's original theory on conventional plates.

From somewhat rewritten Eqs. (13) and (14) it becomes

$$\Big[\frac{Et}{2(1-\nu^2)}\nabla^2 - \frac{G'}{h}\Big]\Big(\frac{\partial r}{\partial x} + \frac{\partial s}{\partial y}\Big) + G'\nabla^2 w = 0 \qquad (16)$$

Since in Eqs. (15) and (16) the terms $\partial r/\partial x + \partial s/\partial y$ appear in a combined form, eliminating them by substitution we obtain a linear differential equation in terms of w only,

$$\Big[\frac{2Eth}{G'(1-\nu^2)} + h^2\Big]\nabla^4 w - 4I_1^m\nabla^2 w + \frac{2(1-\nu^2)}{Et}q = 0 \qquad (17)$$

On the contrary, if we eliminate w from Eqs. (15) and (16), we get

$$\Big\{-\Big[\frac{Et}{G'(1-\nu^2)}I_1^m + \frac{h}{2}\Big]\nabla^2 + \frac{2}{h(1-\nu^2)}I_1^m\Big\}\Big(\frac{\partial r}{\partial x}$$

$$+ \frac{\partial s}{\partial y}\Big) + \frac{1-\nu^2}{Et}q = 0 \qquad (18)$$

Boundary conditions are expressed as usual.[6]

Example of Analysis

As an illustration of the previous derived approximate governing equations, we consider a bending of a supported rectangular sandwich plate $(a\times b)$ with constraints in inplane displacements at the boundaries. The boundary conditions are expressed as follows

$$\text{at } x=0 \text{ and } a,\ w=0,\ M_x=0,\ u^u+u^\ell=0,\ v^u+v^\ell=0,\ s=0 \qquad (19a)$$

at $y=0$ and b, $w=0$, $M_y=0$, $u^u+u^l=0$, $v^u+v^l=0$, $r=0$ (19b)

where M_x and M_y denote bending moments.

We assume w, r and s, which must satisfy Eqs. (19), as follows

$$w=\bar{w}\sin(\pi x/a)\sin(\pi y/b) \tag{20a}$$

$$r=\bar{r}\cos(\pi x/a)\sin(\pi y/b) \tag{20b}$$

$$s=\bar{s}\sin(\pi x/a)\cos(\pi y/b) \tag{20c}$$

Substituting Eqs. (20) into Eqs. (13) and (14), we obtain simultaneous equations with respect to $\bar{r}$ and $\bar{s}$, which are solved as follows

$$\bar{r}=\bar{\bar{r}}\,\bar{w}, \quad \bar{s}=\bar{\bar{s}}\bar{w} \tag{21}$$

where

$$\bar{\bar{r}}=\frac{G'\pi}{ad}\left[\frac{Et\pi^2}{4(1+\nu)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{G'}{h}\right] \tag{22a}$$

$$\bar{\bar{s}}=\frac{G'\pi}{bd}\left[\frac{Et\pi^2}{4(1+\nu)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{G'}{h}\right] \tag{22b}$$

$$d=\frac{Et\pi^2}{4(1-\nu^2)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\left[\frac{Et\pi^2}{2(1+\nu)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)+\frac{G'(3-\nu)}{h}\right]+\frac{G'^2}{h^2} \tag{22c}$$

From Eq. (15) we derive the following equation in the case of sinusoidal distribution of the lateral load $q=\bar{q}\sin(\pi x/a)\times\sin(\pi y/b)$,

$$-\left(\frac{2Et}{1-\nu^2}I_l^m+G'h\right)\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\pi^2\bar{w}+G'\pi\left(\frac{\bar{r}}{a}+\frac{\bar{s}}{b}\right)+\bar{q}=0 \tag{23}$$

I_l^m included in the previous equations is constant owing to Eqs. (11) and (12). The defining Eq. (6) of I_l^m is integrated, over the whole domain of the plate under the boundary condtions (19), to give

$$I_l^m=\tfrac{1}{8}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)\pi^2\bar{w}^2 \tag{24}$$

By introducing Eq. (24) into Eq. (23) and rearranging terms, we obtain the following cubic algebraic equation in terms of $\bar{w}$.

$$\frac{Et\pi^4}{4(1-\nu^2)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2\bar{w}^3+G'h\pi\times\left[\pi\left(\frac{1}{a^2}+\frac{1}{b^2}\right)-\frac{1}{h}\left(\frac{\bar{\bar{r}}}{a}+\frac{\bar{\bar{s}}}{b}\right)\right]\bar{w}-\bar{q}=0 \tag{25}$$

In the isotropic plate case, fundamental equations for large deflection of orthotropic sandwich plates by Nowinski and Ohnabe[6] are reduced to the same ones obtained earlier by Reissner[1] for isotropic sandwich plates. They are expressed in terms of the deflection and a stress function, and correspond to the von Karman type equations for a single-layer plate. For the present example, obeying the fundamental field equations of Reissner or Nowinski-Ohnabe and employing the same displacement functions (20), we get the following equations for $\bar{w}$

$$\frac{Et\pi^4}{4(1-\nu^2)}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2F\left(\frac{b}{a}\right)\bar{w}^3+G'h\pi\times\left[\pi\left(\frac{1}{a^2}+\frac{1}{b^2}\right)-\frac{1}{h}\left(\frac{\bar{\bar{r}}}{a}+\frac{\bar{\bar{s}}}{b}\right)\right]\bar{w}-\bar{q}=0 \tag{26}$$

where

$$F\left(\frac{b}{a}\right)=1+\tfrac{1}{2}(1-\nu^2)\left[\frac{1-(b/a)^2}{1+(b/a)^2}\right]^2-(1-\nu)^2\frac{(b/a)^2}{[1+(b/a)]^2} \tag{27}$$

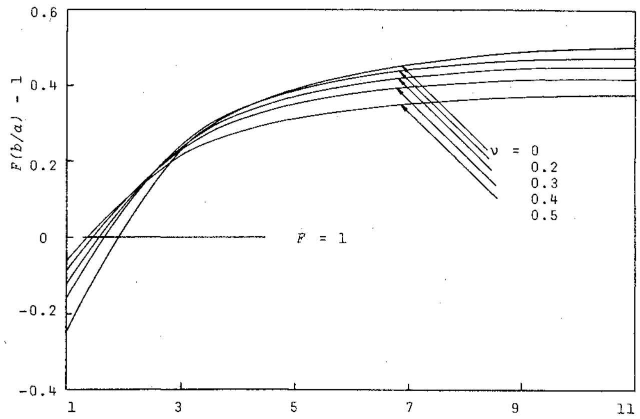

Fig. 1 Correction function.

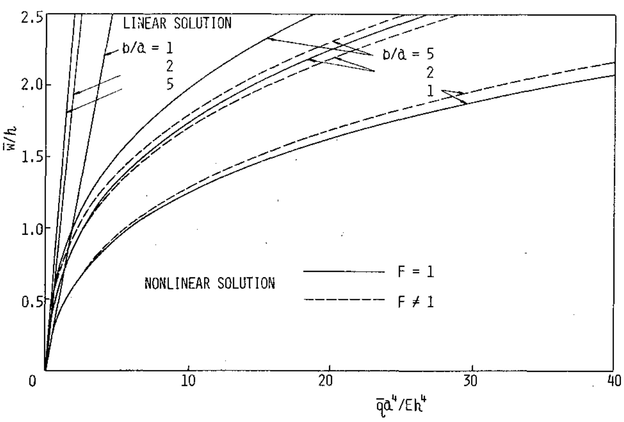

Fig. 2 Maximum deflection of sandwich plate.

It is natural that the coefficients of $\bar{w}$ are identical in Eqs. (25) and (26). The difference between Eqs. (25) and (26) is only in the coefficients of $\bar{w}^3$ and this can be offset by a correction function $F(b/a)$. If $F(b/a)=1$, both equations are perfectly identical. $F(b/a)$ depends on the Poisson's ratio of the faces and on the geometrical parameter of the plate b/a (Fig. 1).

Figure 2 illustrates numerical calculation results of the maximum deflections of rectangular plates obtained by Eqs. (25) and (26), where geometries of plates and the material constants are identical to those utilized in the investigation of Nowinski-Ohnabe, $a=10$ in, $t=0.025$ in, $h=0.6746$ in, $E=10.45\times10^6$ psi, $G'=6\times10^3$ psi, and $\nu=0.3$.

This figure indicates that the accuracy of the results depends directly on the aspect ratio b/a of plate: that is, for plates of b not considerably different from a, the present approximations are fairly good, but if b is large compared with a, the difference becomes correspondingly large.

References

[1] Reissner, E., "Finite Deflections of Sandwich Plates," *Journal of Aeronautical Sciences*, Vol. 15, July 1948, pp. 435-440.

[2] Wang, C. T., "Principle and Application of Complementary Energy Method for Thin Homogeneous and Sandwich Plates and Shells with Finite Deflections," NACA TN, 2620, 1952, pp. 1-33.

[3] Alwan, A. M., "Large Deflection of Sandwich Plates with Orthotropic Cores," *AIAA Journal*, Vol. 2, Oct. 1964, pp. 1820-1822.

[4] Wempner, G. A., and Baylor, J. L., "General Theory of Sandwich Plates with Dissimilar Facings," *International Journal of Solids and Structures*, Vol. 1, 1965, pp. 157-177.

[5] Sharifi, P., and Popov, E. P., "Nonlinear Finite Element Analysis of Sandwich Shells of Revolution," *AIAA Journal*, Vol. 11, May 1973, pp. 715-723.

[6] Nowinski, J. L., and Ohnabe, H., "Fundamental Equations for Large Delections of Sandwich Plates with Orthotropic Core and Faces," *Proceedings of 10th International Symposium on Space Technology and Sciences*, Tokyo, 1973, pp. 311-318.

[7] Berger, H. M., "A New Approach to the Analysis of Large Deflections of Plates," *Journal of Applied Mechanics*, Vol. 22, Dec. 1955, pp. 465-472.

[8] Iwinski, T., and Nowinski, J. L., "The Problem of Large Deflections of Orthotropic Plates (I)," *Archium Mechaniki Stosowanej*, Vol. 9, 1957, pp. 593-603.

[9] Nowinski, J. L., and Ismail, I. A., "Certain Approximate Analyses of Large Deflections of Cylindrical Shells," *Zeitschrift für angewandte Mathematik und Physik*, Vol. 15, 1964, pp. 449-456.

[10] Nash, W. A. and Modeer, J. R., "Certain Approximate Analyses of the Nonlinear Behavior of Plates and Shallow Shells," *Proceedings of Symposium on Theory of Thin Elastic shells*, Delft, 1959, Interscience, N.Y., 1960.

Incompressible Unsteady Flow through a Tube of Variable Cross-Section

D.P. Flemming*

Data Logic Canada Ltd., Ottawa, Ontario

A relationship connecting the rate of change of the volume flow rate of an inviscid incompressible fluid through a tube of varying cross-section with the pressure difference between two arbitrary cross-sections, should prove useful in hydraulic applications and has been used to simulate piston extrusion in a hypervelocity launcher.

The flow is assumed to take place through a uni-directional tube whos axis will be taken as the x axis, and whose cross-section area is given by a suitably differentiable quantity $A(x)$. In addition to incompressibility and zero viscosity, the absence of gravity and other body forces is assumed. While the varying cross-section area prevents the situation from being purely one-dimensional, we use the "quasi-one-dimensional" approach of Rudinger[1] assuming that: a) the column of fluid is long in relation to its cross-section, b) the cross-section area A is a slowly varying function of x.

The flow velocity u and the pressure, p, are thus assumed to be functions of x and the time t. To derive the relation under discussion† we use the Eqs. of conservation of mass and momentum:

$$(Au)_x = 0; Au = f(t) \tag{1}$$

$$u_t + uu_x + Vp_x = 0 \tag{2}$$

where V is the (constant) specific volume.
Thus

$$u_t = f'(t)/A(x) = -(\tfrac{1}{2}u^2)_x - Vp_x \tag{3}$$

Integrating (3) between 2 arbitrary cross-sections, x_1 and x_2, with t fixed, gives

$$f'(t)\int_{x_1}^{x_2}\frac{dx}{A(x)} = -\tfrac{1}{2}(u_2^2-u_1^2)+V(p_1-p_2)$$
$$= -\tfrac{1}{2}f(t)^2[A_2^{-2}-A_1^{-2}]+V(p_1-p_2) \tag{4}$$

the subscripts 1 and 2 denoting values at x_1 and x_2, respectively. Solving (4) for $f'(t)$

$$f'(t) = \frac{V(p_1-p_2)-\tfrac{1}{2}f(t)^2[A_2^{-2}-A_1^{-2}]}{\displaystyle\int_{x_1}^{x_2}\frac{dx}{A(x)}} \tag{5}$$

The previous Eq. has been found useful in the simulation of light gas hypervelocity launchers, specifically the simulation of the extrusion of a plastic piston, after reaching a region of decreasing cross-section area. While conditions a) and b) previously mentioned may not be satisfied, this approach was felt to give a reasonable approximation as part of the total launcher simulation.

Clearly Eq. (5) can be used to determine the piston motion from the pressures at its 2 ends. This in turn allows the use of the same Eq. for determining the pressure at intermediate points. The possibility of obtaining negative internal pressures should be born in mind. This would probably result in the longitudinal separation of the material.

Equation (5) might also be applied to transient problems in hydraulics. In the case of steady flow, in which $f'(t) \equiv 0$. Eq. (5) given the Bernouilli relation,

$$Vp_1 + \tfrac{1}{2}u_1^2 = Vp_2 + \tfrac{1}{2}u_2^2 \tag{6}$$

Reference

[1] Rudinger, George., *Wave Diagrams for Unsteady Flow in Ducts*, D.Van Nostrand, N.Y., 1955.

Received June 4, 1975; revision received September 25, 1975. This paper is based on research with which the author was associated while employed at the Defence Research Establishment Valcartier (formerly CARDE), Courcelette, Quebec, Canada. Their consent to its publication is gratefully acknowledged.

Index categories: LV/M Simulation; Hydrodynamics; Nozzle and Channel Flow.

*Consulting Analyst, Applied Sciences Group.

†The relation (equation (5)) is believed to have been originated by the late Mario Cloutier at DREV-who developed it for the light-gas gun application discussed below. The author is grateful to the Associate Editor for the particular derivation given here.

Numerical Investigation of Leading-Edge Vortex for Low-Aspect Ratio Thin Wings

Colmar Rehbach*

Office National d'Etudes et de Recherches Aerospatiales (ONERA), Chatillon, France

I. Introduction

FOR plane and cambered delta wings and for wings with curved leading edges exhibiting the well-known leading-edge vortex flow phenomenon,[1-5] we present theoretical results obtained by a singularities method.[6] These results are compared with flow visualizations performed in a water-tunnel.

Received June 23, 1975; revision received Oct. 21, 1975.

Index category: Subsonic and Transonic Flow.

*Research Scientist, Aerodynamics Department.